Infinite Limit of R/P Ratio and Rate of Depletion

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Part One : US Multilogistic Modelisation

Hubbert's Multilogistic Modelisation of the Oil Exploitation Cycle, concerning a whole system of oil deposit, is very efficient. In the case of the USA, the three graphs below make visible that the multilogistic modelisation is very similar to the "past" lines. It allows to estimate the future path of oil exploitation. The US multilogistic modelisation has been made with the sum of three logistic cycles. It is possible to extend the modelisation to much more cycles. The logistic modelisation of each production's cycle and each discoveries' cycle is led by three factors : Q_o (ultimate reserves), t_o (production's peak) and a (growth factor).

Here are the constants of the three cycles for production and discoveries: Ultimate Reserves = 224 Gb ; Cumulative discoveries in 2005 = 213,8 Gb ; ; Cumulative Production in 2005 = 192 Gb (89,8 % of cumulative disc.and 85,7 % of Ultimate Reserves); Reserves Rr in 2005 = 21,5 Gb ; Reserves Ru in 2005 = 31,7 Gb ;
The first oil exploitation cycle is the conventional oil production in the 48 US States : Cycle P1 : Q_o = 197 Gb ; t_o = 1972 ; a = 0,06 ; Cycle D1 : Q_o = 197 Gb ; t_o = 1940 ; a = 0,06 ;
The second oil exploitation cycle is the oil production in the Alaska : Cycle P2 : Q_o = 15 Gb ; t_o = 1985 ; a = 0,15 ; Cycle D2 : Q_o = 15 Gb ; t_o = 1969 ; a =1,5 ;
and the third is the oil production in the Gulf Of Mexico : Cycle P3 : Q_o = 12 Gb ; t_o = 2006 ; a = 0,2 ; Cycle D3 : Q_o = 12 Gb ; t_o = 2003 ; a = 0,25.

GRAPH 1

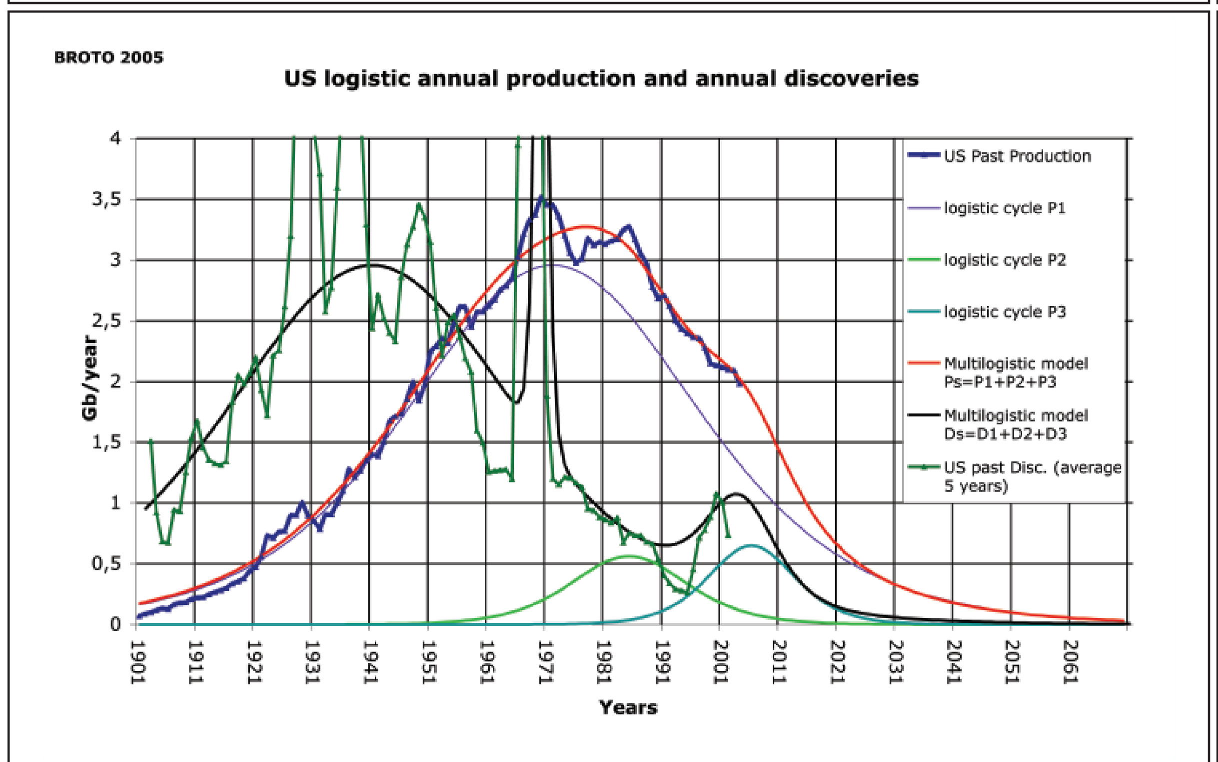
On the graph 1 "the modelisation of the annual production and annual discoveries", the green line shows the past discoveries for the US (on average of 5 years) . The dark line is the modelisation of the major discoveries coming from the 48 US states, Prudhoe Bay (Alaska) and Gulf Of Mexico's. There is also the logistic lines of cycle P1, P2 and P3 and their sum as multilogistic model (Ps) visible in red line. The blue line represents the US past production.

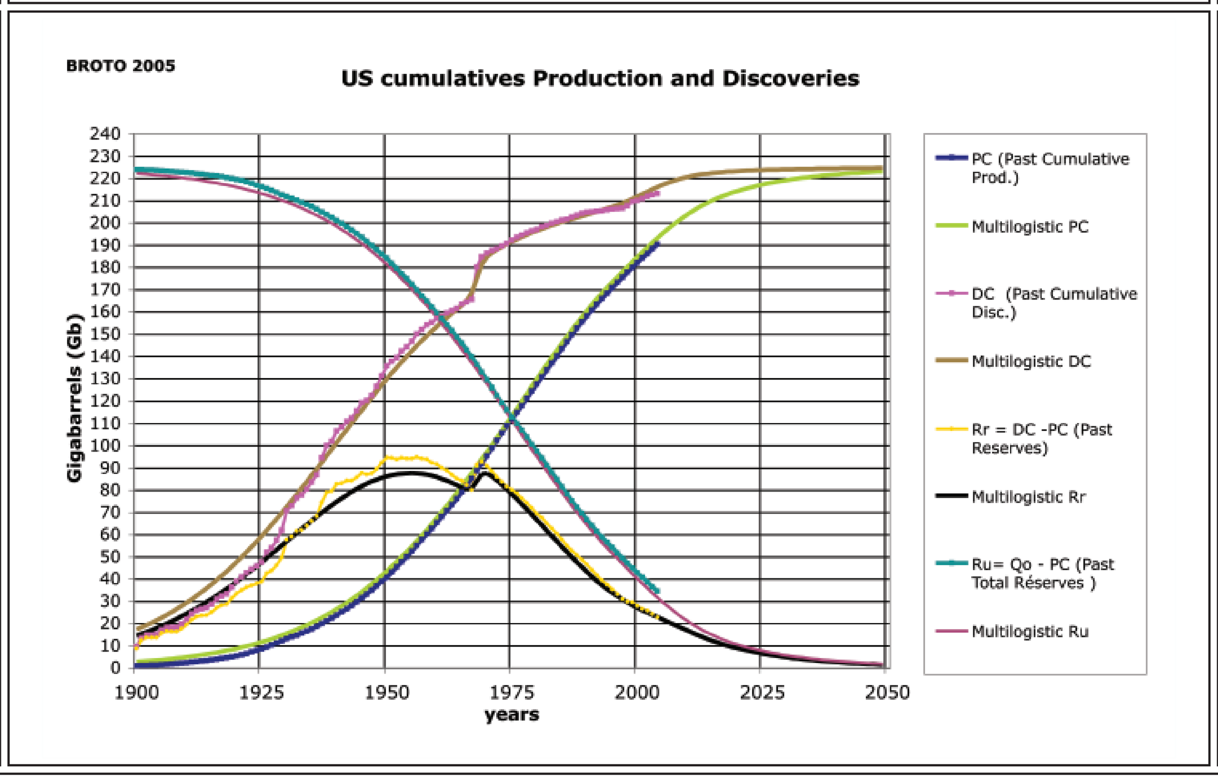
GRAPH 2

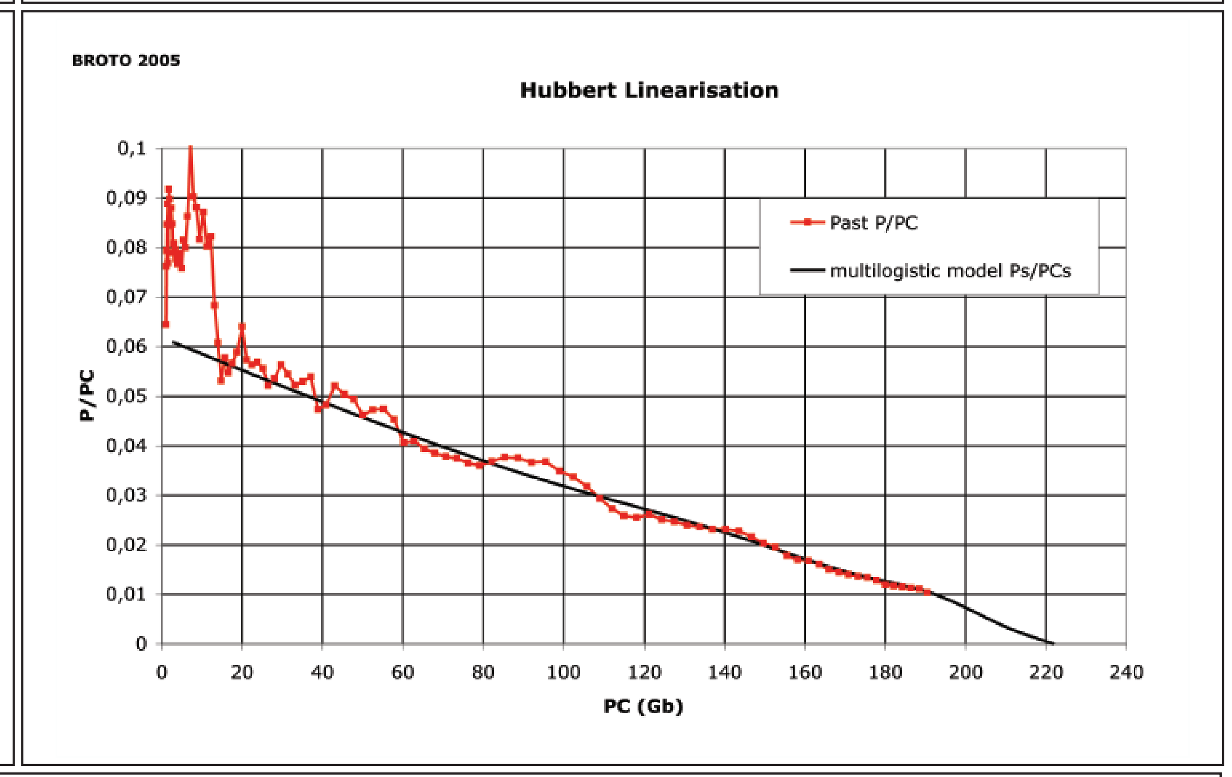
Graph 2 shows the US cumulative production (PC) and discoveries (DC) lines, the ultimate reserves left to produce (Ru) and the reserves left to produce from the discoveries already done (Rr).
We have the past and the multilogistic lines for PC, DC, Ru and Rr. You can see that theory and reality are very similar in this graph.
In this model, the ultimate discovery of the US is 224 Gb. The US has already produced 192 Gb in 2005 which represent 85,7% of the ultimate discovery. They have already discovered 213,8 Gb, which left 10,2 Gb to discover in the future. We can see that the whole production cycle of the USA is very mature in 2005.

GRAPH 3

Graph 3 describe the famous linearisation line of Hubbert's model for the USA. That graph describe the percentage of cumulative production growth each year (P/PC) versus the cumulative production. This graph is very interesting because the line become linear after the first period of drilling, so that you can extrapolate ultimate reserves. The past and logistic line are nearly the same on this graph. The multilogistic model leads to the fact that the line is not really linear and is much more closer to the past curve, especially at the end of the past line.







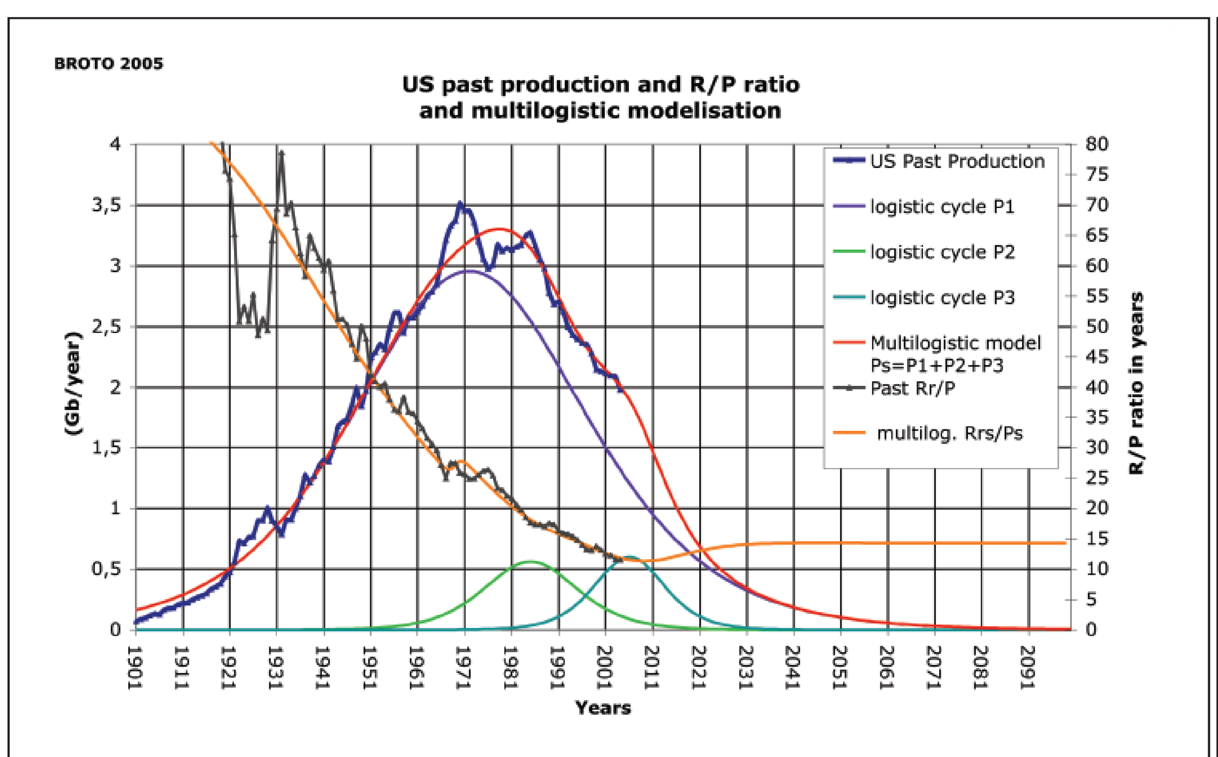
Part Two : Infinite Limit of R/P Ratio and Static and Political Rate of Depletion

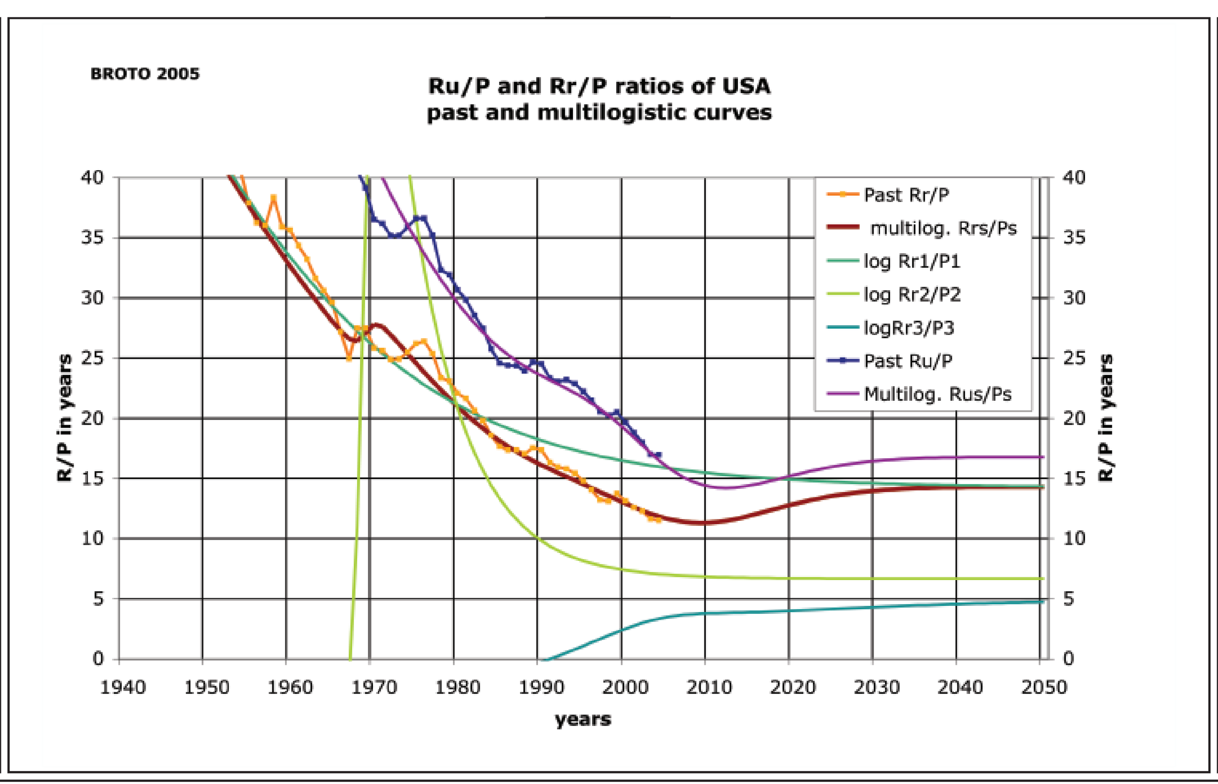
Infinite Limit of R/P Ratio

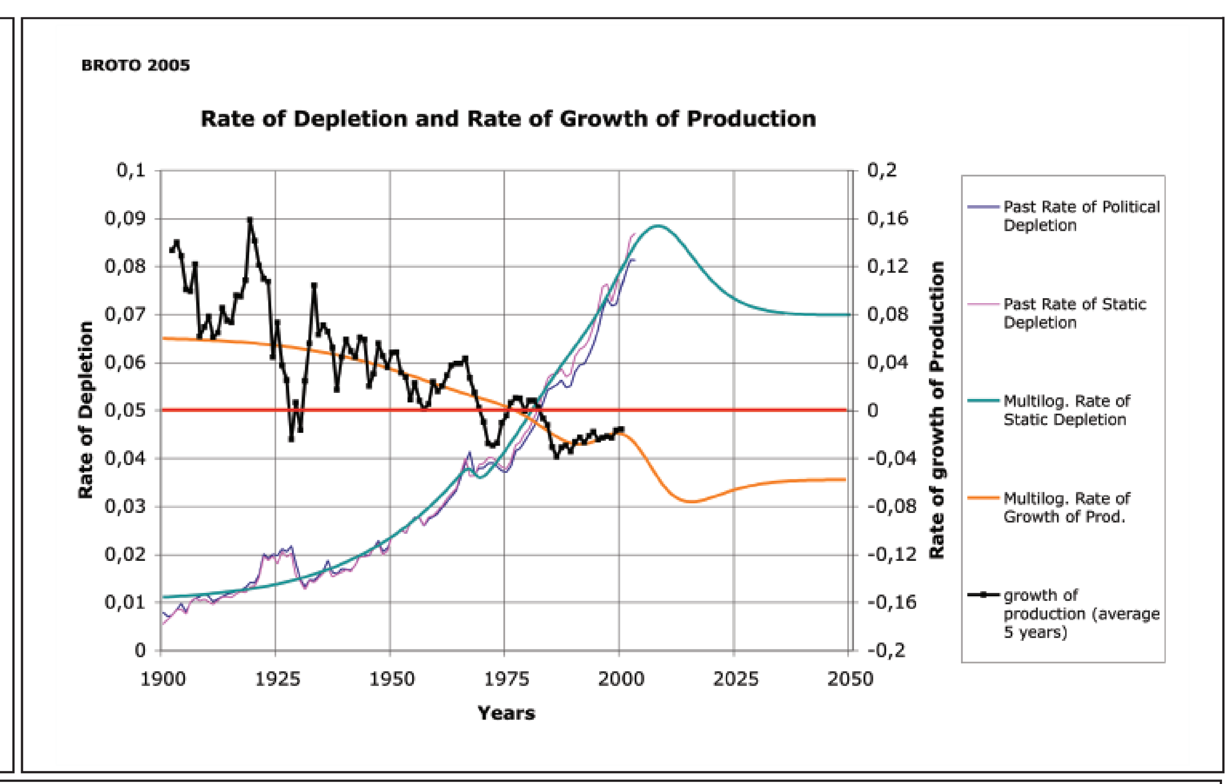
Here I describe the infinite limit of R/P with the Hubbert modelisation. The R/P is well-known by the oil industry and by the public. It is the most common and oldest model of oil prediction. The R/P ratio makes possible the calculation of the number of remaining years of current production versus reserves. But this is not that accurate to follow this model. I've tried to calculate the R/P with the Hubbert Logistic Equations and I discovered that theses equations lead the ratio R/P toward a numerical limit when times goes to the infinite. So the R/P ratio never goes to 0. This discovery leads me to conclude that the Hubbert's equations are going to put definitely the ratio R/P to dustbin. But, I discovered that this infinite limit was very interesting because of the relationship between the R/P ratio and Colin Campbell's Rate of Depletion. So, the fact that the R/P ratio has a limit leads to the fact that the rate of Depletion and the rate of Growth of Production both have a numerical limit at the end of the exploitation. After that, I researched a mathematical way to calculate this limit and I find it for one logistic cycle but also for a multilogistic model. So, I can calculate the infinite limit of R/P ratio in all possible mulilogistic modelisation. That's a fact that the growth factor "a" of the logistic equation is very important in this calcul.
You can use the ultimate reserves Ru or the discovery reserves Rr to find two different limits as you can see in the center graph below for the US. For example, the US infinite limit of the Ru/P is 16,6 years and the US infinite limit of the Rr/P is 14,2 years. The past and the multilogistic R/P are also very close on the graphs. In the case of the US, the fact that the logistic model show that Rr/P goes back up to 14 after touching the lowest number, 11,3 years, in 2009, could seem weird, but the reason is that third cycle (GOM) is based on a very high growth factor "a", which is 0,2. The center graph below shows that the multilogistic Rr/P line goes to the infinite limit of the logistic Rr1/P1 line. Because the first cycle of the US 48 has the lowest "a" growth factor comparing to the two others.

Static and Political Rate of Depletion

I call Colin Campbell's Rate of Depletion : "The Political Rate of Depletion" or "Tp" because it is very useful in a political way, as you can modulate the production of the coming year according to the Rate of Depletion you want and last year's Reserves .
For the year n, the equation of the Political Rate of Depletion is Tpn = Pn/Rn-1.
"The Static Rate of Depletion" or "Ts" is exactly the reverse of the R/P. It is useful in the technical way for the Protocol Depletion in the P(R) graph as you can see in the next part. The Political Rate of Depletion is the Static Rate of Depletion minus the Rate of Growth of the Reserves.
As you can see on the graph, the difference between the two Rates is not very important and doesn't go above 1%. For example, the US Political Rate of Depletion is 8,13% in 2004 and the Static Rate of Depletion is 8,69% in 2004.
So, the Static and Political Rate of Depletion (PRD) goes toward an infinite limit in the same way as the ratio R/P. In the case of the US, the infinite limit of the Static Rate of Depletion (SRD) is 7%. The meaning of this limit is that the Static Rate of Depletion goes toward this Rate as the Production goes to an end, even if it is above it for a moment.
I call the infinite limit of the Static Rate of Depletion the "Nominal Static Rate of Depletion" (NSRD). I call the infinite limit of the Political Rate of Depletion the "Nominal Political Rate of Depletion" (NPRD). Here, the NSRD of the US is 7% and is the NSRD of the cycle1 (US 48). You can see that the multilogistic SRD (green line) will peak in 2009 at 8,83% and after will decline toward 7%.
I also study the Rate of Growth of the Production (orange line) which has a infinite limit at 6%. The Rate of Growth is about to plunge toward 8%/year when the SRD will arrive at his peak.







Part Three : P(R) Graph and Dynamic Rate of Depletion

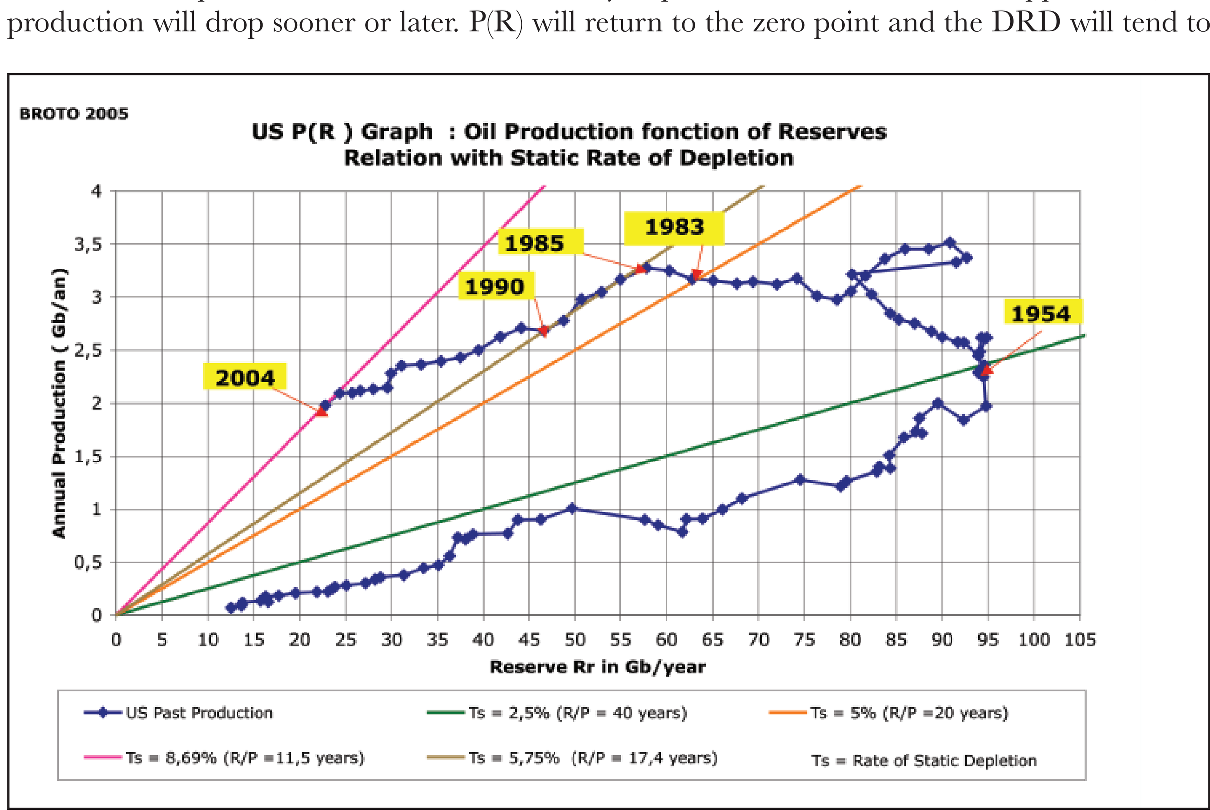
P(R) Graph and Dynamic Rate of Depletion

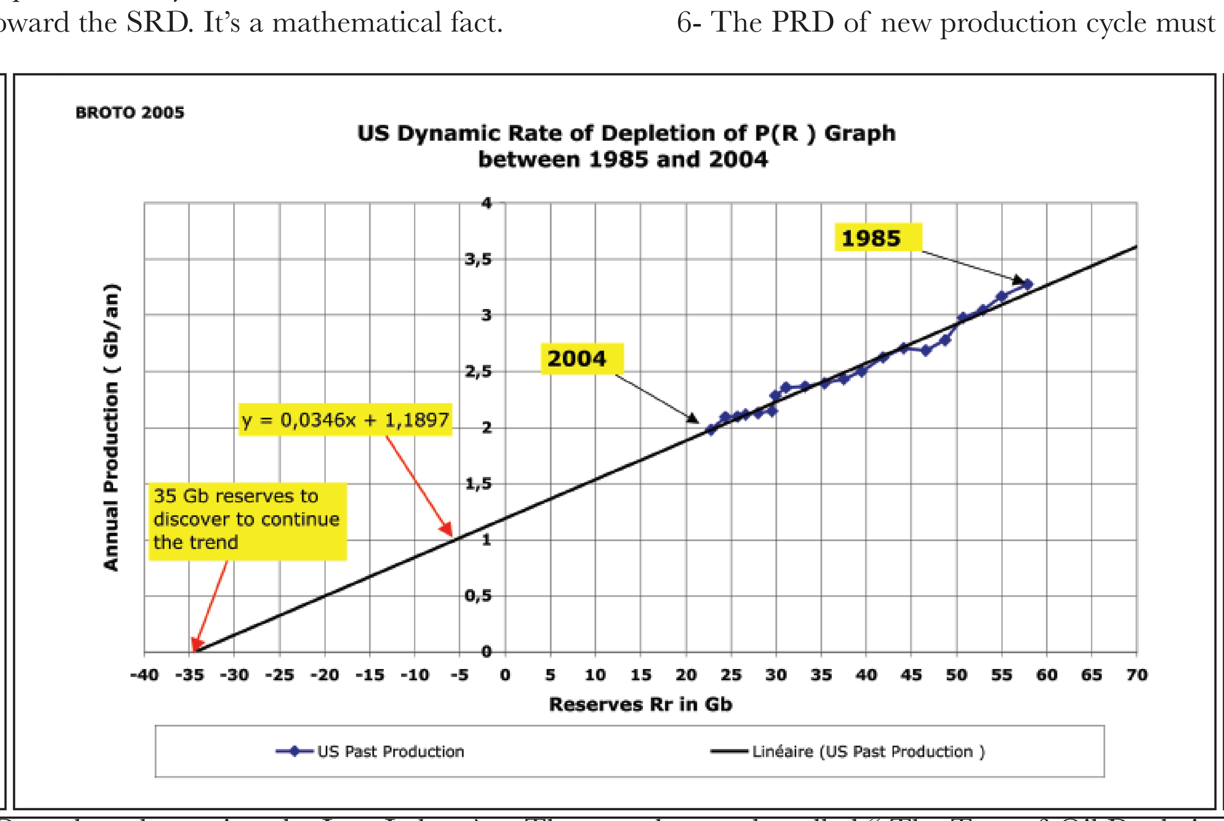
The graph P(R) shows the Production P versus the Reserves Rr. This Graph has some advantages : the line begins at the Zero point and comes to an end at the same point, so you know the beginning and the end. You can see on the line the evolution of different periods and peak points of the whole cycle : the discovery period ; the growth of production period ; the peak of the Reserves when the annual production overshoots the annual discoveries ; the period between the reserves' peak and the production's peak ; the production decline period. So, you have lots of informations in the same graph. This graph is also interesting because ot its relationship with the Static Rate of Depletion (SRD) as you can see in the graph below. Each point of the P(R) line, which corresponds to one year, crosses a straight line coming from the zero point. The slope of the straight line is the SRD. So each year, the P(R) line crosses another straight line and takes another SRD. So, The SRD doesn't give informations about the relationship between two points of the P(R) line but only about the situation of one point (in one area of the graph). It's the reason why I call this notion "Static Rate of Depletion". In this graph, the SRD represents the Political Rate of Depletion (PRD), the Campbell's Rate of Depletion. It's the same for the PRD.
That's why I think that the SRD or the PRD is not enough to see what's happening in the production cycle and I propose another notion, the DYNAMIC RATE OF DEPLETION (DRD) .
The Dynamic Rate of Depletion is the slope of the straight regression line of a part of P(R) line, which is also called "the straight line of the lesser squares". For example, in the period between 1985 and 2004 for the US, the slope of the straight regression line of the P(R) is 3,46% (center graph below) but the SRD began at 5,75% in 1985 and ended at 8,69% in 2004. In the center graph below, you can see that the US need to discover another 35 Gb to continue this trend (pushing the ultimate toward 248 Gb instead of 224Gb), which seems very unlikely. The relationship between SRD and DRD is very important because, if DRD is upper SRD, the production cycle in not sustainable and the production will drop sooner or later. P(R) will return to the zero point and the DRD will tend toward the SRD. It's a mathematical fact.

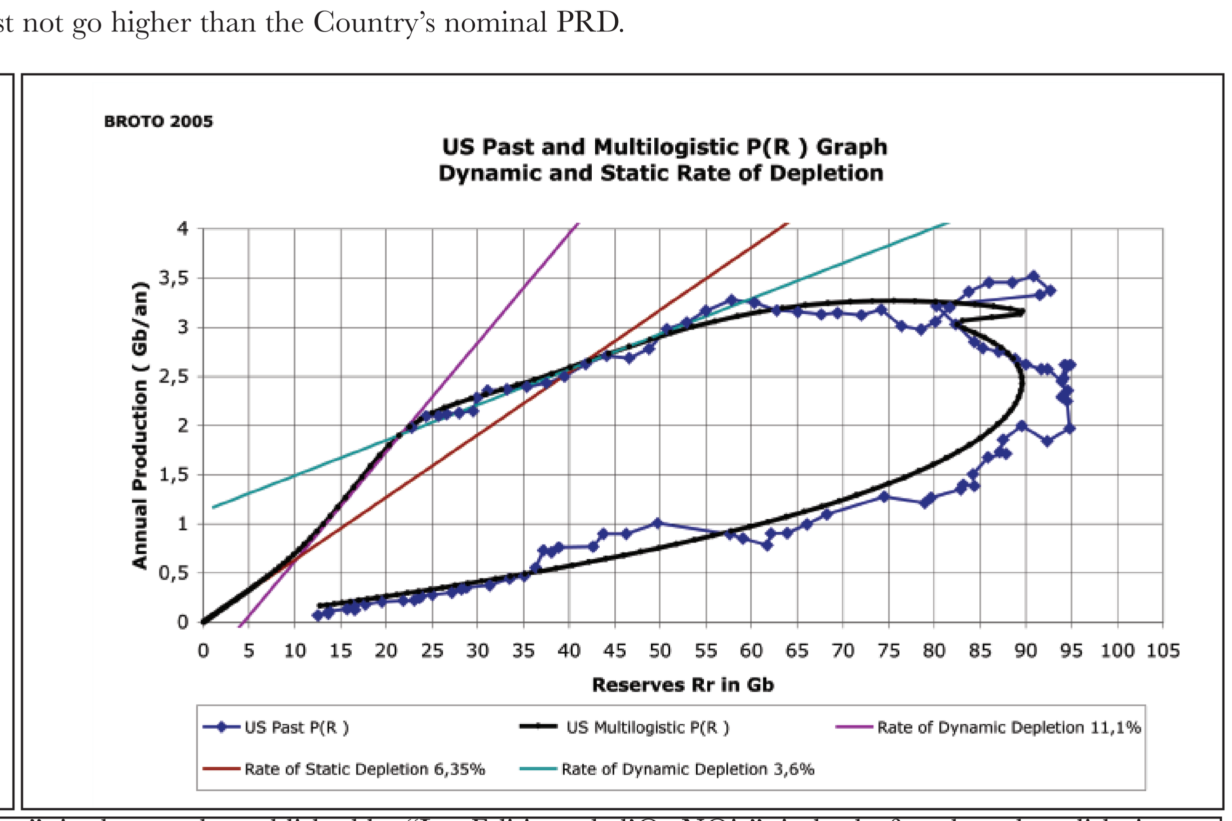
Proposals for the Depletion Protocol on the side of producers countries

Colin Campbell has proposed the Depletion Protocol to avoid the most destructive effects of the peak oil on the human society. I support this will and the Depletion Protocol. I would like to propose some ideas about how to determinate the quotas of production within the Depletion Protocol. Here are my proposals :
1- The Depletion Protocol must audit the reserves and production in signatories countries ;
2- The multilogistic modelisation of oil discoveries and production cycle becomes the reference model of the Depletion Protocol ;
3- The Nominal Political Rate of Depletion (NPRD) is calculated for each production country ;
4- With the P(R) graph, the relationship between Dynamic Rate of Depletion (DRD)and Static Rate of Depletion (SRD) determines what has to be done :
- If DRD is higher or equal than current SRD, then the production cycle is sustainable.
  - If the current SRD is higher than Nominal SRD, then the country must lower his current SRD to Nominal SRD;
  - If the current SRD is lower or equal than Nominal SRD, then the country can push the production to the Nominal SRD
- If DRD is lower than SRD, then the production cycle in not sustainable (over-production) ;
  - The first period : the DRD must go up to the current SRD (consequences : the production must drop)
  - The second period : the current SRD must lower to the Nominal SRD of the country.

After appraisal has been made on P(R) graph, SRD is always translated in PRD to establish the production quota of countries for the coming year.
5- Funds are created for countries which reach their Nominal PRD.
6- The PRD of new production cycle must not go higher than the Country's nominal PRD.







This study has been made by Emmanuel Broto in 2005-2006 with the help of Jean Laherrère. Datas have been given by Jean Laherrère. The complete study, called " The Trap of Oil Depletion", is about to be published by "Les Editions de l'Or NOir" in both french and english, in September 2006.
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Poster made for the V conference of Association for Study of Peak Oil and Gas in 18th and 19th of july 2006 in Pisa.